



Class 11 Maths || Chapter 4 : Principle of Mathematical Induction ||

Class Notes

Toppers Village

Class-11- Maths

chapter-4

Principle of Mathematical Induction

PMI

Deductive Thinking

Inductive Reasoning

e.g.

$$1+2+3+\dots+n = \frac{n(n+1)}{2}$$

$\{1, 2, 3, \dots\}$

$n \in \mathbb{N}$

series

formula (sum)

$P(n)$

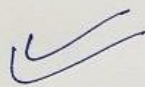
In this chapter: ^{only} 4.1

- 3 types
- Equation $1 \rightarrow 17$ ^{c.g.} 1, 3, 8
 - Divisibility $19 \rightarrow 23$ 4, 6
 - Inequality $18, 24$ 2, 5, 7

How to prove using PMI

$P(n)$: Statement

- (I) $P(1)$ Prove
- (II) Let $P(k)$ is true
then prove that
 $P(k+1)$ is also true.



Exercise 4.1 PMI

$$(i) \quad 1 + 3 + 3^2 + \dots + 3^{n-1} = \frac{3^n - 1}{2}, n \in \mathbb{N}$$

Proof using PMI

$$P(n): 1 + 3 + 3^2 + \dots + 3^{n-1} = \frac{3^n - 1}{2}$$

I-step "we have to prove that $P(1)$ is true."

$$P(1): 1 = \frac{3^1 - 1}{2}$$

$$\Rightarrow 1 = \frac{2}{2} \quad \therefore P(1) \text{ is true.}$$

$$\Rightarrow 1 = 1$$

$$\boxed{\text{LHS} = \text{RHS}}$$

II-step: let $P(k)$ is true.

$$1 + 3 + 3^2 + \dots + 3^{k-1} = \frac{3^k - 1}{2} \quad \text{--- (1)}$$

(1)

$$\textcircled{3^0 = 1}$$

Now we will have to prove that $P(k+1)$ is also true.

$$P(k+1): 1 + 3 + 3^2 + \dots + 3^{(k+1)-1} = \frac{3^{k+1} - 1}{2}$$

$$\text{LHS of } P(k+1) = \underbrace{1 + 3 + 3^2 + \dots + 3^{k-1}}_{\text{--- (1)}} + 3^k$$

$$= \frac{3^k - 1}{2} + 3^k$$

$$= \frac{3^k - 1 + 2 \cdot 3^k}{2}$$

$$= \frac{3^1 \cdot 3^k - 1}{2} = \frac{3^{k+1} - 1}{2}$$

$$= \text{RHS of } P(k+1)$$

$\therefore P(k+1)$ is also true.

$\therefore P(n)$ is true (proved using PMI)

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Class-11- Maths Exercise 4.1

(PMI)

Q.2 $1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$

Proof:

$P(n): 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$

I-Step: " $P(1)$ is true" $\leftarrow$ Prove.

$$P(1): 1^3 = \left[\frac{1(1+1)}{2} \right]^2$$

$$\Rightarrow 1 = 1 \quad \therefore P(1) \text{ is true.}$$

II-Step: Let " $P(k)$ is true"

$$P(k): 1^3 + 2^3 + 3^3 + \dots + k^3 = \left[\frac{k(k+1)}{2} \right]^2 \quad \text{--- (1)}$$

Now we have to prove that $P(k+1)$ is true.

$$P(k+1): 1^3 + 2^3 + 3^3 + \dots + (k+1)^3 = \left[\frac{(k+1)(k+2)}{2} \right]^2$$

LHS of $P(k+1)$

$$\begin{aligned} &= \underbrace{1^3 + 2^3 + 3^3 + \dots + k^3}_{\left[\frac{k(k+1)}{2} \right]^2} + \underbrace{(k+1)^3}_{(k+1)^3} \\ &= \left[\frac{k(k+1)}{2} \right]^2 + (k+1)^3 \end{aligned}$$

$$= (k+1)^2 \cdot \left\{ \frac{k^2}{4} + k+1 \right\}$$

$$= (k+1)^2 \cdot \left\{ \frac{k^2 + 4k + 4}{4} \right\} = \frac{(k+1)^2 \cdot (k+2)^2}{4}$$

$$= \left(\frac{(k+1) \cdot (k+2)}{2} \right)^2$$

= RHS of $P(k+1)$

$\therefore P(k+1)$ is true. ✓

$\therefore P(n)$ is true (using PMI).

Q.3

$$P(n): 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+n} = \frac{2n}{n+1}$$

Proof: I - step: Prove $P(1)$ is true

$$\Rightarrow P(1): 1 = \frac{2 \times 1}{1+1}$$

$$\Rightarrow 1 = \frac{2}{2}$$

$$\Rightarrow 1 = 1 \quad \therefore P(1) \text{ is true. } \checkmark$$

II - step: let $P(k)$ is true.

$$P(k): 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+3+\dots+(k)} = \frac{2(k)}{(k)+1}$$

Now we have to prove that $P(k+1)$ is also true.

$$P(k+1): 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+\dots+(k+1)} = \frac{2(k+1)}{(k+1)+1}$$

$$\text{LHS of } P(K+1) = 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \dots + \frac{1}{1+2+\dots+K} + \frac{1}{1+2+3+\dots+(K+1)}$$

$$= \left(\frac{2K}{K+1} \right) +$$

$$\frac{1}{\underbrace{1+2+3+\dots+K+(K+1)}_{\substack{\text{AP.} \\ a \quad l}}}$$

Sum of AP.

$$S_n = \frac{n}{2} (a+l)$$

$$S_{K+1} = \frac{K+1}{2} (1+K+1)$$

$$\begin{aligned} a &= 1 \\ d &= 1 \end{aligned}$$

$n = \text{no. of terms}$

$$= K+1$$

$$= \left(\frac{K+1}{2} \cdot (K+2) \right)$$

$$= \frac{2K}{K+1} + \frac{1}{\left(\frac{K+1}{2} \right) \cdot (K+2)}$$

$$= \left(\frac{2K}{K+1} \right) + \frac{2}{(K+1) \cdot (K+2)}$$

$$= \frac{2K(K+2) + 2}{(K+1) \cdot (K+2)}$$

$$= \frac{2 \cdot \{ K^2 + 2K + 1 \}}{(K+1)(K+2)}$$

$$= \frac{2 \{ K+1 \}^2}{(K+1) \cdot (K+2)}$$

$$= \frac{2(K+1)}{(K+2)} = \text{RHS of } P(K+1)$$

$\therefore P(n)$ is true (using PMI)

Exercise 4.1 PM I : Toppers Village

Q. 4
$$P(n) : \overbrace{1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + n(n+1)(n+2)} = \frac{n(n+1)(n+2)(n+3)}{4}$$

Proof: I-step: " $P(1)$ is true" $\leftarrow$ To prove.

$$P(1): 1 \cdot 2 \cdot 3 = \frac{1(2)(3)(\cancel{4})}{\cancel{4}}$$

$$\Rightarrow 6 = 6$$

$\therefore P(1)$ is true. $\checkmark$

II-step: Let $P(k)$ is true.

$$P(k): \overbrace{1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + k(k+1)(k+2)} = \frac{k(k+1)(k+2)(k+3)}{4} \quad \text{--- (1)}$$

~~Now~~ Now we have to prove that $P(\underline{k+1})$ is also true.

$$P(k+1): \underline{1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + (k+1)(k+2)(k+3)} = \underline{\frac{(k+1)(k+2)(k+3)(k+4)}{4}}$$

$$\text{LHS of } P(k+1) = 1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + (k+1)(k+2)(k+3)$$

$$= \underbrace{1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + k(k+1)(k+2)}_{\downarrow} + (k+1)(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$$

$$= (k+1)(k+2)(k+3) \cdot \left\{ \frac{k}{4} + 1 \right\}$$

$$= \frac{(k+1)(k+2)(k+3) \cdot (k+4)}{4}$$

$$= \text{RHS of } P(k+1)$$

$\therefore P(n)$ is true (Using PMI)

Q.5 $P(n): \underbrace{1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \dots + n \cdot 3^n}_{4} = \frac{(2n-1) \cdot 3^{n+1} + 3}{4}$

Proof: I - step: $P(1)$ is true.

$$P(1): 1 \cdot 3 = \frac{(2 \times 1 - 1) \cdot 3^{1+1} + 3}{4}$$

$$\Rightarrow 3 = \frac{1 \times 9 + 3}{4} = \frac{12}{4}$$

$$\Rightarrow 3 = 3 \quad \therefore P(1) \text{ is true.}$$

II - step: $P(k)$ is true (Assume)

$$P(k): \underbrace{1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \dots + k \cdot 3^k}_{4} = \frac{(2k-1) \cdot 3^{k+1} + 3}{4} \quad \text{--- (1)}$$

Now we have to prove that $P(k+1)$ is also true.

$$P(k+1): 1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \dots + \underline{(k+1) \cdot 3^{k+1}} = \frac{[2(k+1)-1] \cdot 3^{k+1+1} + 3}{4}$$

$$= \frac{(2k+1) \cdot 3^{k+2} + 3}{4}$$

$$\text{LHS of } P(K+1) = \underbrace{1 \cdot 3 + 2 \cdot 3^2 + 3 \cdot 3^3 + \dots + K \cdot 3^K}_{\substack{4 \\ (2K-1) \cdot 3^{K+1} + 3}} + (K+1) \cdot 3^{K+1}$$

$$= \frac{(2K-1) \cdot 3^{K+1} + 3}{4} + (K+1) \cdot 3^{K+1}$$

$$= \frac{(2K-1) \cdot 3^{K+1} + 3 + 4(K+1) \cdot 3^{K+1}}{4}$$

$$= \frac{[2K-1 + 4K+4] 3^{K+1} + 3}{4}$$

$$= \frac{(6K+3) 3^{K+1} + 3}{4}$$

$$= \frac{(2K+1) \cdot 3 \cdot 3^{K+1} + 3}{4}$$

$$= \frac{(2K+1) \cdot 3^{K+2} + 3}{4} = \frac{[2(K+1)-1] \cdot 3^{(K+1)+1} + 3}{4} = \text{RHS of } P(K+1)$$

$\therefore P(n)$ is true (PMI)

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Class-11-Maths

Exercise 4.1

$$\boxed{\text{Q.6}} \quad P(n): \underline{1.2} + \underline{2.3} + \underline{3.4} + \dots + \underline{n(n+1)} = \frac{n(n+1)(n+2)}{3}$$

Proof: I-Step: $\textcircled{P(1)}$ true??

$$P(1): 1.2 = \frac{1 \times 2 \times \cancel{3}}{\cancel{3}}$$

$\Rightarrow 2 = 2 \quad \therefore P(1) \text{ is true.}$

II-Step: let $P(k)$ is true.

$$P(k): \boxed{1.2 + 2.3 + 3.4 + \dots + k(k+1) = \frac{k(k+1)(k+2)}{3}} \quad \textcircled{1}$$

Now = we have to prove that $P(k+1)$ is also true.

$$P(k+1): \underline{1.2 + 2.3 + 3.4 + \dots + (k+1)(k+2)} = \frac{(k+1)(k+2)(k+3)}{3}$$

To prove,

$$\text{LHS of } P(K+1) = 1.2 + 2.3 + 3.4 + \dots + (K+1)(K+2)$$

$$= \underbrace{1.2 + 2.3 + 3.4 + \dots + K.(K+1)} + (K+1)(K+2)$$

$$= \frac{K(K+1)(K+2)}{3} + \underline{(K+1)(K+2)}$$

$$= (K+1)(K+2) \cdot \left\{ \frac{K}{3} + 1 \right\}$$

$$= (K+1)(K+2) \left\{ \frac{K+3}{3} \right\}$$

$$= \frac{(K+1)(K+2)(K+3)}{3}$$

$$= \text{RHS of } P(K+1)$$

$\therefore P(K+1)$ is also true.

$\therefore P(n)$ is true (PMI)

Q.7 $P(n): 1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots + (2n-1)(2n+1) = \frac{n(4n^2 + 6n - 1)}{3}$

I-Step: $P(1)$ is true.

$$\Rightarrow P(1): 1 \cdot 3 = \frac{1(4 + 6 - 1)}{3}$$

$$\Rightarrow 3 = \frac{9}{3}$$

$$\Rightarrow 3 = 3 \quad \therefore P(1) \text{ is true.}$$

II-Step: Let $P(k)$ is true.

$$P(k): \boxed{1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots + (2k-1)(2k+1) = \frac{k(4k^2 + 6k - 1)}{3}} \quad \text{--- (1)}$$

Now we have to prove " $P(k+1)$ is true."

$$P(k+1): 1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots + (2(k+1)-1)(2(k+1)+1) = \frac{(k+1)(4(k+1)^2 + 6(k+1) - 1)}{3}$$

$$\Rightarrow \underline{\underline{1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots + (2k+1)(2k+3) = \frac{(k+1)(4k^2 + 14k + 9)}{3}}}$$

LHS.

$$\text{LHS of } P(K+1) = 1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots + (2K+1)(2K+3)$$

$$= \underbrace{1 \cdot 3 + 3 \cdot 5 + 5 \cdot 7 + \dots + (2K+1)(2K+1)}_{\substack{\uparrow \\ K(4K^2 + 6K - 1)}} + \underbrace{(2K+1) \cdot (2K+3)}_{\substack{\uparrow \\ (2K+1)(2K+3)}}$$

$$= \frac{K(4K^2 + 6K - 1)}{3} + (2K+1)(2K+3)$$

$$= \frac{4K^3 + 6K^2 - K + 3(4K^2 + 8K + 3)}{3}$$

$$= \frac{4K^3 + 18K^2 + 23K + 9}{3}$$

$$= \frac{4K^3 + 18K^2 + 23K + 9}{3}$$

$(K+1)$ factor

$$= \frac{(K+1)(4K^2 + 14K + 9)}{3} = \frac{(K+1)(4(K+1)^2 + 6(K+1) - 1)}{3} = \text{RHS of } P(K+1)$$

$$\begin{array}{r} 4K^3 + 14K + 9 \\ K+1 \overline{) 4K^3 + 18K^2 + 23K + 9} \\ \underline{4K^3 + 4K^2} \\ 14K^2 + 23K \\ \underline{14K^2 + 14K} \\ 9K + 9 \\ \underline{9K + 9} \\ 0 \end{array}$$

$$\begin{array}{l} K = -1 \\ -4 + 18 - 23 + 9 = 0 \end{array}$$

$$\textcircled{8} \quad P(n) = \underbrace{1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + n \cdot 2^n}_{= (n-1) \cdot 2^{n+1} + 2}$$

I-Step:

$$P(1): 1 \times 2 = \underbrace{(1-1) \cdot 2^{1+1}}_{= 0} + 2$$

$$\Rightarrow 2 = 2$$

$\therefore P(1)$ is true. $\checkmark$

II-Step:

Let $P(k)$ is true.

$$P(k): \underbrace{1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + \cancel{k \cdot 2^k}}_{= (k-1) \cdot 2^{k+1} + 2} = (k-1) \cdot 2^{k+1} + 2 \quad \text{--- } \textcircled{1}$$

Now we have to prove that $P(k+1)$ is also true.

$$\begin{aligned} \therefore P(k+1): & \underbrace{1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + (k+1) \cdot 2^{k+1}}_{= (k+1-1) \cdot 2^{k+1+1} + 2} \\ & = \underline{\underline{k \cdot 2^{k+2} + 2}} \end{aligned}$$

$$\text{LHS of } P(K+1) = \underbrace{1 \cdot 2 + 2 \cdot 2^2 + 3 \cdot 2^3 + \dots + K \cdot 2^K}_{\downarrow} + (K+1) \cdot 2^{K+1}$$

$$= (K-1) \cdot \underbrace{2^{K+1}}_{\text{circled}} + 2 + (K+1) \cdot \underbrace{2^{K+1}}_{\text{circled}}$$

$$= \left(\overset{K-1}{\cancel{+ K+1}} \right) 2^{K+1} + 2$$

$$= 2 \cdot K \cdot 2^{K+1} + 2$$

$$= K \cdot 2^{K+2} + 2$$

$$= (\underline{K+1} - 1) \cdot 2^{\underline{K+1} + 1} + 2$$

$$= \text{RHS of } P(K+1)$$

$\therefore P(n)$ is true (PMI)

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Class-11-Maths Exercise 4.1 (PMI)

[Q.9] $P(n): \underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n}} = 1 - \frac{1}{2^n}$

I-Step: To prove "P(1) is true"

$$P(1): \frac{1}{2} = 1 - \frac{1}{2^1}$$

$$\Rightarrow \frac{1}{2} = \frac{1}{2} \quad \therefore P(1) \text{ is true.}$$

II-Step: Let $P(K)$ is true.

$$P(K): \underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^K}} = 1 - \frac{1}{2^K} \quad \text{--- (1)}$$

Now we have to prove that $P(K+1)$ is also true.

$$P(K+1): \underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^{K+1}}} = \left(1 - \frac{1}{2^{K+1}}\right)$$

LHS of $P(K+1)$

$$= \underbrace{\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^K}} + \frac{1}{2^{K+1}}$$

$$= \underbrace{1 - \frac{1}{2^K}} + \frac{1}{2^{K+1}}$$

$$= 1 - \underbrace{\frac{2}{2^{K+1}} + \frac{1}{2^{K+1}}}$$

$$= 1 + \frac{-2 + 1}{2^{K+1}}$$

$$= \left(1 - \frac{1}{2^{K+1}}\right)$$

= RHS. of $P(K+1)$

$\therefore P(n)$ is true
(PMI)

$$\textcircled{10} \quad P(n): \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{(3n-1)(3n+2)} = \frac{n}{6n+4}$$

I-Step. "P(1) is true"

$$\underline{P(1)} = \frac{1}{2 \cdot 5} = \frac{1}{6 \times 1 + 4}$$

$$\Rightarrow \frac{1}{10} = \frac{1}{10} \quad \therefore P(1) \text{ is true.}$$

II-STEP: Let $P(k)$ is true.

$$P(k): \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{(\underline{3k-1})(\underline{3k+2})} = \frac{\overset{\textcircled{1}}{k}}{6k+4}$$

Now we have to prove that $P(k+1)$ is true.

$$P(k+1): \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{\{3(k+1)-1\} \cdot \{3(k+1)+2\}} = \frac{k+1}{6(k+1)+4}$$

$$\Rightarrow \left(\frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{(3k+2)(3k+5)} \right) = \frac{k+1}{6k+10} \quad ??$$

$$\text{LHS of } P(K+1) = \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{(3K+2)(3K+5)}$$

$$= \frac{1}{2 \cdot 5} + \frac{1}{5 \cdot 8} + \frac{1}{8 \cdot 11} + \dots + \frac{1}{(3K-1)(3K+2)} + \frac{1}{(3K+2)(3K+5)}$$

$$= \frac{K}{6K+4} + \frac{1}{(3K+2)(3K+5)}$$

$$= \frac{K}{2(3K+2)} + \frac{1}{(3K+2)(3K+5)}$$

$$= \frac{3K^2 + 5K + 2}{2(3K+2)(3K+5)}$$

$$= \frac{3K^2 + 3K + 2K + 2}{2(3K+2)(3K+5)}$$

$$= \frac{3K(K+1) + 2(K+1)}{2(3K+2)(3K+5)}$$

$$= \frac{(K+1)(3K+2)}{2(3K+2)(3K+5)}$$

$$= \frac{K+1}{6K+10}$$

$$= \text{R.H.S. of } P(K+1).$$

$\therefore P(n)$ is true
(By PMI)

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Class-11-Maths Exercise 4.1 (PMI)

Q.11 $P(n): \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{n(n+1)(n+2)} = \frac{n(n+3)}{4(n+1)(n+2)}$

I-Step: " $P(1)$ is true"

$$P(1): \frac{1}{1 \cdot 2 \cdot 3} = \frac{1(\cancel{4})}{\cancel{4} \cdot (2)(3)}$$

$$\frac{1}{6} = \frac{1}{6} \quad \therefore P(1) \text{ is true.}$$

II-Step: (let $P(k)$ is true.)

$$P(k): \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{k(k+1)(k+2)} = \frac{k(k+3)}{4(k+1)(k+2)} \quad \text{--- (1)}$$

Now we have to prove that $P(k+1)$ is also true.

$$P(k+1): \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{(k+1)(k+2)(k+3)} = \frac{(k+1)(k+4)}{4(k+2)(k+3)}$$

$$\text{LHS of } P(k+1) = \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \dots + \frac{1}{k(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{k(k+3)}{4(k+1)(k+2)} + \frac{1}{(k+1)(k+2)(k+3)}$$

$$= \frac{1}{(k+1)(k+2)} \cdot \left\{ \frac{k(k+3)}{4} + \frac{1}{k+3} \right\}$$

$$= \frac{1}{(k+1)(k+2)} \cdot \left\{ \frac{k(k^2 + 6k + 9) + 4}{4(k+3)} \right\} \quad \left(\frac{k^3 + k^2}{k^2(k+1)} \right)$$

$$= \frac{1}{(k+1)(k+2)} \cdot \left\{ \frac{k^3 + 6k^2 + 9k + 4}{4(k+3)} \right\}$$

$(k = -1)$ root $\rightarrow$ $(k+1)$ factor

$$= \frac{1}{(k+1)(k+2)} \cdot \left\{ \frac{k^3 + k^2 + 5k^2 + 5k + 4k + 4}{4(k+3)} \right\}$$

$$\begin{aligned} &= \frac{1}{(k+1)(k+2)} \cdot \left\{ \frac{k^2(k+1) + 5k(k+1) + 4(k+1)}{4(k+3)} \right\} \\ &= \frac{(k+1)(k^2 + 5k + 4)}{(k+1)(k+2) \cdot 4(k+3)} \\ &= \frac{\cancel{(k+1)} \cdot (k+1)(k+4)}{(\cancel{k+1})(k+2) \cdot 4 \cdot (k+3)} \\ &= \frac{(k+1)(k+4)}{4(k+2) \cdot (k+3)} \\ &= \text{RHS of } P(k+1) \\ &\therefore \underline{\underline{P(n) \text{ is true.}}} \end{aligned}$$

$$(12) P(n): a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

I-step: "P(1) is true"

$$P(1): a = \frac{a(r^1 - 1)}{(r - 1)}$$

$\Rightarrow \boxed{a=a} \therefore P(1) \text{ is true.}$

II-step: let P(k) is true.

$$P(k) = \underbrace{a + ar + ar^2 + \dots + a \cdot r^{(k)-1}}_{\text{---}} = \underbrace{\frac{a(r^{(k)} - 1)}{r - 1}}_{\text{---}}$$

Now we have to prove that P(k+1) is true.

$$\underline{P(k+1)}: a + ar + ar^2 + \dots + \underbrace{a \cdot r^{(k+1)-1}}_{\substack{\downarrow \\ a \cdot r^k}} = \frac{a(r^{k+1} - 1)}{r - 1}$$

$$\text{LHS of } P(k+1) = \underbrace{a + ar + ar^2 + \dots + a.r^{k-1}} + a.r^k$$

$$= \frac{a.(r^k - 1)}{r - 1} + a.r^k$$

$$= \frac{\cancel{a.r^k} - a + a.r^k.r^1 - \cancel{a.r^k}}{(r-1)}$$

$$= \frac{a(r^{k+1} - 1)}{(r-1)}$$

$$= \text{RHS of } P(k+1)$$

$\therefore P(n)$ is true (By using PMI)

class-11-mathsExercise 4.1 (PMI)

Q.13 $P(n): \left(1 + \frac{3}{1}\right) \cdot \left(1 + \frac{5}{4}\right) \cdot \left(1 + \frac{7}{9}\right) \cdots \left(1 + \frac{2n+1}{n^2}\right) = (n+1)^2$

I. Step: $P(1)$ is true.

$$P(1): \left(1 + \frac{3}{1}\right) = (1+1)^2$$

$$\Rightarrow 4 = 4 \quad \therefore P(1) \text{ is true.}$$

II. Step: ~~$P(k)$~~ let $P(k)$ is true.

$$P(k): \underbrace{\left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \cdots \left(1 + \frac{2k+1}{k^2}\right)} = (k+1)^2 \quad \text{--- (1)}$$

Now we ~~are~~ will have to prove that $P(k+1)$ is also true.

$$\text{* } P(k+1): \underbrace{\left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \cdots \left(1 + \frac{2k+3}{(k+1)^2}\right)} = (k+2)^2$$

$$\text{LHS of } P(K+1) = \underbrace{\left(1 + \frac{3}{1}\right) \left(1 + \frac{5}{4}\right) \dots \left(1 + \frac{2K+1}{K^2}\right)}_{\swarrow} \cdot \left(1 + \frac{2K+3}{(K+1)^2}\right)$$

$$= \underbrace{(K+1)^2}_{\swarrow} \cdot \left\{ \frac{(K+1)^2 + 2K+3}{\cancel{(K+1)^2}} \right\}$$

$$= K^2 + 2K + 1 + 2K + 3$$

$$= \frac{K^2 + 4K + 4}{}$$

$$= (K+2)^2$$

$$= \text{RHS. of } P(K+1).$$

$\therefore P(n)$ is true (By PMI)

$$\textcircled{14} P(n): \left(1 + \frac{1}{1}\right) \left(1 + \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \dots \left(1 + \frac{1}{n}\right) = (n+1)$$

I-Step: P(1)

$$P(1): \left(1 + \frac{1}{1}\right) = 1+1$$

$\Rightarrow 2 = 2 \therefore P(1)$ is true.

II-Step: let $P(k)$ is true.

$$P(k): \left(1 + \frac{1}{1}\right) \left(1 + \frac{1}{2}\right) \left(1 + \frac{1}{3}\right) \dots \left(1 + \frac{1}{k}\right) = (k+1) \quad \textcircled{1}$$

Now we will have to prove that $P(k+1)$ is also true.

$$P(k+1): \underbrace{\left(1 + \frac{1}{1}\right) \left(1 + \frac{1}{2}\right) \dots \left(1 + \frac{1}{k+1}\right)}_{\text{LHS of } P(k+1)} = \underbrace{(k+2)}_{\text{RHS of } P(k+1)}$$

$$\text{LHS of } P(k+1) = \underbrace{\left(1 + \frac{1}{1}\right) \left(1 + \frac{1}{2}\right) \dots \left(1 + \frac{1}{k}\right)}_{\text{LHS of } P(k)} \cdot \left(1 + \frac{1}{k+1}\right)$$

$$= \cancel{(k+1)} \cdot \left(\frac{k+1+1}{\cancel{k+1}}\right) = (k+2) = \text{RHS of } P(k+1)$$

$\therefore \underline{\underline{P(n) \text{ is true.}}}$

$$(15) p(n): 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

I. Step: $p(1)$

$$p(1): 1^2 = \frac{1(1)(3)}{3}$$

$\Rightarrow 1 = 1 \therefore p(1)$ is true.

II. Step: Let $p(k)$ is true. ①

$$p(k): 1^2 + 3^2 + 5^2 + \dots + (2k-1)^2 = \frac{k(2k-1)(2k+1)}{3}$$

Now we will have to prove that ~~$p(k)$~~ $p(k+1)$ is also true.

$$p(k+1): 1^2 + 3^2 + 5^2 + \dots + (2(k+1)-1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$$

$$1^2 + 3^2 + 5^2 + \dots + (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$$

~~~~~  
 $\therefore$



$$\text{LHS of } P(k+1) = \underbrace{1^2 + 3^2 + 5^2 + \dots + (2k-1)^2}_{\swarrow} + (2k+1)^2$$

$$= \frac{k(2k-1)(2k+1)}{3} + \underline{(2k+1)^2}$$

$$= (2k+1) \cdot \left\{ \frac{k(2k-1)}{3} + \frac{(2k+1)}{1} \right\}$$

$$= (2k+1) \cdot \left\{ \frac{2k^2 - k + 6k + 3}{3} \right\}$$

$$= (2k+1) \cdot \left\{ \frac{2k^2 + 5k + 3}{3} \right\}$$

$$= (2k+1) \frac{(2k^2 + 2k + 3k + 3)}{3}$$

$$= \frac{(2k+1)(2k(k+1) + 3(k+1))}{3}$$

$$= \frac{(2k+1)(k+1)(2k+3)}{3}$$

$$= \underline{\text{RHS. of } P(k+1)}$$

$\therefore \underline{\underline{P(n) \text{ is true.}}}$

Toppers Village

Class-11-Maths    Exercise 4.1 (PMI)

Q.16. 
$$P(n): \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \frac{1}{7 \cdot 10} + \dots + \frac{1}{(3n-2) \cdot (3n+1)} = \frac{n}{3n+1}$$

I-step: "P(1) is true"

$$P(1): \frac{1}{1 \cdot 4} = \frac{1}{4} \quad \therefore P(1) \text{ is true.}$$

II-step: Let  $P(k)$  is true.  $\implies P(k): \boxed{\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} = \frac{k}{3k+1}}$   
"P(k+1) is also true"  $\leftarrow$  To prove. (1)

$$\underline{P(k+1)}: \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{[3(k+1)-2] \cdot [3(k+1)+1]} = \frac{k+1}{3(k+1)+1}$$

$$\Rightarrow \left\{ \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3k+1) \cdot (3k+4)} = \frac{k+1}{3k+4} \right\} ??$$



$$\text{LHS of } P(k+1) = \frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \dots + \frac{1}{(3k-2)(3k+1)} + \frac{1}{(3k+1)(3k+4)}$$

LHS of  $P(k)$

$$= \underbrace{\frac{k}{3k+1}} + \frac{1}{(3k+1)(3k+4)}$$

$$= \frac{3k^2 + 4k + 1}{(3k+1) \cdot (3k+4)}$$

$$= \frac{(\cancel{3k+1})(k+1)}{(\cancel{3k+1}) \cdot (3k+4)}$$

$$= \frac{k+1}{3k+4} = \text{RHS of } P(k+1)$$

$\therefore P(k+1)$  is true.

$\therefore P(n)$  is true.

(PMT)

$$(17) \quad P(n): \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \dots + \frac{1}{\underset{\uparrow}{(2n+1)} \cdot \underset{\uparrow}{(2n+3)}} = \frac{n}{3(2n+3)} \quad \leftarrow$$

(I) ~~P~~ "P(1) is true"

$$P(1): \frac{1}{3 \times 5} = \frac{1}{3 \times 5} \therefore P(1) \text{ is true.}$$

(II) let  $P(K)$  is true:

$$P(K): \left[ \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2K+1) \cdot (2K+3)} \right] = \frac{K}{3(2K+3)} \quad \text{--- (1)}$$

To Prove - " $P(K+1)$  is also True"

$$P(K+1): \underbrace{\frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2K+3)(2K+5)}}_{\substack{\uparrow \\ \text{To prove}}} = \frac{K+1}{3(2K+5)}$$

To prove



$$\text{LHS of } P(K+1) = \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2K+1)(2K+3)} + \frac{1}{(2K+3)(2K+5)}$$

$$= \frac{K}{3(2K+3)} + \frac{1}{(2K+3) \cdot (2K+5)}$$

$$= \frac{2K^2 + 5K + 3}{3(2K+3) \cdot (2K+5)} \quad \begin{array}{r} 2K^2 + 2K + 3K + 3 \\ \hline 2K \qquad 3 \end{array}$$

$$= \frac{(2K+3)(K+1)}{3(2K+3) \cdot (2K+5)}$$

$$= \frac{K+1}{3(2K+5)} = \text{RHS of } P(K+1)$$

$\therefore P(K+1)$  is true.

---

$\therefore P(n)$  is true.

# Toppers Village

Class-11-Maths. Exercise 4.1 (PMI)

Q.19

Q.18, Q.24 → Last

$P(n)$ :  $n(n+1)(n+5)$  is a multiple of 3.

I-step To prove —  $P(1)$  is true.

$P(1)$ :  $1 \cdot 2 \cdot 6$  is a multiple of '3'  
 $\Rightarrow 12$  is a multiple of '3' ✓  
 $\therefore$   $P(1)$  is true.

(II) - step: let  $P(k)$  is true.

$P(k)$ :  $k(k+1)(k+5)$  is a multiple of '3'  $\Rightarrow$   $k(k+1)(k+5) = 3 \times m$

" $P(k+1)$  is also true" ← To prove.

$\hookrightarrow P(k+1)$ :  $(k+1) \cdot (k+2) \cdot (k+6)$  is a multiple of '3'  
 $\Rightarrow$   $(k+1) \cdot (k+2) \cdot (k+6) = 3 \times q$ ,  $q \in \mathbb{N}$

Divisible, multiple, factor

$6 = 3 \times 2$   
6 is a multiple of 3

factor  
↓

①

,  $m \in \mathbb{N}$



Assumed  $P(K)$

$$K(K+1)(K+5) = 3.m \quad (1)$$

$$\Rightarrow K(K^2 + 6K + 5) = 3m$$

$$\Rightarrow \boxed{K^3 + 6K^2 + 5K = 3m}$$

(2)

To Prove  $P(K+1)$

$$(K+1)(K+2)(K+6) = 3.9$$

$$\text{LHS} = (K+1)(K+2)(K+6)$$

$$= (K+1) \cdot \{K^2 + 8K + 12\}$$

$$= K^3 + 8K^2 + 12K + K^2 + 8K + 12$$

$$= K^3 + 9K^2 + 20K + 12$$

$$= (\underline{K^3 + 6K^2 + 5K}) + 3K^2 + 15K + 12$$

$$= \underline{3.m + 3K^2 + 15K + 12}$$

$$= 3 \cdot \underline{\{m + K^2 + 5K + 4\}}$$

$$= 3.9 = \text{RHS of } P(K+1)$$

## Toppers Village:

Q.20  $P(n): \underbrace{10^{2n-1} + 1}$  is divisible by 11.

I-step:

$P(1): \underbrace{10^{2 \times 1 - 1} + 1}$  is divisible by 11.

$\Rightarrow 11$  is divisible by 11.

$P(1) \longrightarrow \boxed{\text{True}}$  ✓

II-step: let  $P(k)$  is true.

$P(k): 10^{2k-1} + 1$  is divisible by 11.  $\Rightarrow 10^{2k-1} + 1 = 11 \cdot m$

$\Rightarrow \frac{10^{2k}}{10} = 11m - 1$

$\Rightarrow \boxed{10^{2k} = (11 \cdot m - 1) \cdot 10} \text{ --- (1)}$

To Prove,  
↓

$P(k+1): 10^{2(k+1)-1} + 1$  is divisible by 11

$\Rightarrow \underline{\underline{10^{2k+1} + 1}}$  is divisible by 11 "



Now,  $10^{2K+1} + 1$

$$= 10^{2K} \cdot 10^1 + 1$$

$$= (11m-1) \underline{10 \cdot 10} + 1$$

By eq<sup>n</sup> (1)

$$= 1100m - 100 + 1$$

$$= 1100m - 99$$

$$= 11 \cdot \{100m - 9\}$$

$$= 11 \cdot 9$$

$10^{2K+1} + 1$  is divisible by '11'  $\leftarrow P(K+1)$

Proved

$\therefore P(n)$  is true (Proved)

# Toppers Village

Class. 11. Maths.

Exercise 4.1 (PMI)

[Q.21]  $P(n): x^{2n} - y^{2n}$  is divisible by ' $x+y$ '

I-Step.  $P(1): x^2 - y^2$  is divisible by ' $x+y$ '

$\Rightarrow$   $(x+y)(x-y)$  is divisible by ' $x+y$ '

(True)

II-Step. Let  $P(k)$  is true

$\Rightarrow P(k): x^{2k} - y^{2k}$  is divisible by ' $x+y$ '

$$\Rightarrow \underline{x^{2k} - y^{2k}} = (x+y).m$$

$$\Rightarrow \boxed{x^{2k} = (x+y).m + y^{2k}} \quad \text{--- (1)}$$

To Prove

$\hookrightarrow P(k+1)$  is also true.

$\therefore P(k+1): \underline{x^{2(k+1)} - y^{2(k+1)}}$  is divisible by ' $x+y$ '.



$$x^{2(K+1)} - y^{2(K+1)}$$

$$= x^{2K} \cdot x^2 - y^{2K} \cdot y^2$$

By eq<sup>n</sup> ①

$$= \left[ \underline{(x+y) \cdot m + y^{2K}} \right] \cdot x^2 - y^{2K} \cdot y^2$$

$$= \underline{x^2 \cdot (x+y) \cdot m} + \underline{x^2 \cdot y^{2K}} - \underline{y^{2K} \cdot y^2}$$

$$= \underline{(x+y) \cdot x^2 \cdot m} + y^{2K} \cdot \{ \underline{x^2 - y^2} \}$$

$$= \underline{(x+y) \cdot x^2 \cdot m} + y^{2K} \cdot \underline{(x+y) \cdot (x-y)}$$

$$= \underline{(x+y)} \cdot \{ x^2 \cdot m + y^{2K} \cdot (x-y) \}$$

$\Rightarrow \therefore x^{2(K+1)} - y^{2(K+1)}$  is divisible by ' $x+y$ '

$\therefore \underline{P(K+1) \rightarrow \text{True}}$

$\therefore \underline{p(n) \text{ is true.}}$

Q. 22  $P(n): \underbrace{3^{2n+2} - 8n - 9}_{\text{is divisible by 8}}$

I - step:  
 $P(1): 3^{2+2} - 8 \times 1 - 9$  is divisible by '8'.  
 $\Rightarrow 81 - 8 - 9$  is divisible by '8'.  
 $\Rightarrow$  '64' is divisible by '8'.  
True

II - step: Let  $P(K)$  is true.

$P(K): 3^{2K+2} - 8K - 9$  is divisible by '8'.

$$\Rightarrow \underbrace{3^{2K+2}}_{\text{is divisible by 8}} - 8K - 9 = 8 \cdot m$$

$$\Rightarrow \boxed{3^{2K+2} = 8m + 8K + 9} \quad \text{--- (1)}$$

To prove:  $P(K+1)$  is true.

$P(K+1): \underbrace{3^{2(K+1)+2} - 8(K+1) - 9}_{\text{is divisible by 8}}$



$$3^{2(k+1)+2} - 8(k+1) - 9$$

$$= \underline{3^{2k+2+2}} - 8(k+1) - 9$$

$$= \underline{3^{2k+2}} \cdot 3^2 - 8(k+1) - 9$$

$$= (8m + 8k + 9)9 - 8k - 8 - 9$$

$$= \underline{8 \cdot 9 \cdot m} + \underline{8 \cdot 9 \cdot k} + \underline{81} - \underline{8k} - \underline{17}$$

$$= 8 \cdot 9 \cdot m + 8 \cdot 9 \cdot k - 8k + 64$$

$$= 8 \cdot \{9m + 9k - k + 8\}$$

$$= 8 \cdot 9 \quad \underline{9 \in \mathbb{N}}$$

$\therefore \cancel{P(k)} P(k+1)$  is true.

$\therefore P(n)$  is true.

Q.23  $P(n)$ :  $41^n - 14^n$  is a multiple of 27. ✓

(I)  $P(1)$ :  $41 - 14$  is a multiple of 27

$\Rightarrow 27$  is a multiple of 27. ✓

$P(1)$  is true.

(II) Let  $P(k)$  is true.

$P(k)$ :  $41^k - 14^k$  is a multiple of 27.

$$\Rightarrow \boxed{41^k - 14^k = 27 \cdot m} \quad \text{--- (1)} \quad \Rightarrow \boxed{41^k = 27 \cdot m + 14^k}$$

To Prove: " $P(k+1)$  is also true."

$P(k+1)$ :  $41^{k+1} - 14^{k+1}$  is a multiple of 27. ✓

$$= \boxed{41^k} \cdot 41 - 14^k \cdot 14$$

$$= (27 \cdot m + 14^k) \cdot 41 - 14^k \cdot 14$$

$$\begin{aligned} &= 27 \cdot 41 \cdot m + \boxed{14^k} \cdot 41 - \boxed{14^k} \cdot 14 \\ &= \underline{27 \cdot 41 \cdot m} + \underline{14^k \cdot (41 - 14)} \\ &= 27 \cdot \{ \underline{41 \cdot m + 14^k} \} \end{aligned}$$

~~$27 \cdot 14^k$~~   $= 27 \cdot 2$



# Toppers Village

Class-11-Maths.

Exercise 4.1 (PMI)

Q.18

Q.24

Inequality

Q.18

$$P(n): 1+2+3+\dots+n < \frac{1}{8}(2n+1)^2$$

(I)

$$P(1): 1 < \frac{1}{8}(3^2)$$

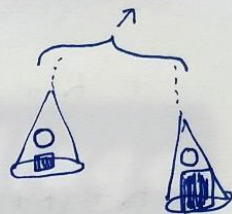
$$\Rightarrow 1 < \frac{9}{8} \text{ (True)}$$

$\therefore P(1)$  is true.

(II)

Let  $P(k)$  is true

$$P(k): 1+2+3+\dots+k < \frac{1}{8}(2k+1)^2 \quad \text{--- (1)}$$



$$\frac{1}{+3} < \frac{5}{+3}$$

$$4 < 8$$

$$\frac{0}{h} > \frac{0}{h} > \frac{0}{h}$$

I II

$$a > b > c$$

$$a > c$$

$$10 > 5 > 2$$

$$10 > 2 ??$$

" $P(K+1)$  is true"  $\leftarrow$  To prove

$$P(K+1): 1+2+3+\dots+K+(K+1) < \frac{1}{8} (2(K+1)+1)^2$$

$$\Rightarrow \underbrace{1+2+3+\dots+K+(K+1)}_{\swarrow} < \frac{1}{8} (2K+3)^2$$

$$\text{LHS} = \underbrace{1+2+3+\dots+K}_{\square} + \frac{(K+1)}{1} < \underbrace{\frac{1}{8} (2K+1)^2}_{\square} + \frac{(K+1)}{1}$$

$$< \frac{4K^2+4K+1+8K+8}{8}$$

$$< \frac{4K^2+12K+9}{8}$$

$$\Rightarrow \frac{1+2+3+\dots+K+(K+1)}{1} < \frac{(2K+3)^2}{8}$$

$\therefore P(K+1)$  is true.

$\therefore \underline{\underline{P(n) \text{ is true.}}}$



Q.24  $P(n): 2n+7 < (n+3)^2$

$P(n): (n+3)^2 > (2n+7)$

$n \in \mathbb{N} \rightarrow 1, 2, 3, \dots$

I-step:  $P(1): (1+3)^2 > (2 \times 1 + 7)$   
 $\Rightarrow 16 > 9$  True.

II-step: let  $P(k)$  is true:  $\Rightarrow P(k): (k+3)^2 > (2k+7)$  inequality — (1)

To prove: " $P(k+1)$  is also true"  $\Rightarrow P(k+1): (k+1+3)^2 > (2(k+1)+7)$   
 $\Rightarrow (k+4)^2 > (2k+9)$

LHS of  $P(k+1) = (k+4)^2$   
 $= (\underline{k+3} + \underline{1})^2$

$= (k+3)^2 + 1 + 2(k+3) \cdot 1$

$= \underline{(k+3)^2} + \underline{2k+7} > \underline{(2k+7)} + \underline{2k+7}$

$100 > 50 > \textcircled{2} ??$

$\begin{array}{c} \underbrace{\hspace{10em}} > \underbrace{\hspace{10em}} \\ \square > \square > \square \end{array}$   
 $\therefore (k+4)^2 > (2k+9)$

## **All Useful Links:**

- **YouTube Channel : Toppers Village** (Get full and free video lectures of Class 10 & 11 Maths) - <https://www.youtube.com/toppersvillage>
- **Website :** [www.toppersvillage.com](http://www.toppersvillage.com)
- **Instagram :** <https://www.instagram.com/toppersvillage/>
- **Facebook Page:** [www.facebook.com/ToppersVillage/](http://www.facebook.com/ToppersVillage/)
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